Math 206A Lecture 27 Notes

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1 Purel's Theorem and Differential Analyzers

1.1 Purel's theorem

Look up a video of a planimeter online.

Proposition 1.1. *There exists a mechanical instrument which "computes" the area of a planar regions.*

Proof. It is sufficient to show that the instrument can measure the area of rectangles. Let's not actually prove this. □

This is a bit different from the mechanical linkages we atalked about last time. This instrument involves a cone and wheels.

Recall Kempe' theorem from last time.

Theorem 1.1 (Kempe, 1880s)**.** *For every semi-algebraic system, there exists a plane linkage which "solves" it.*

By a semi-algebraic system, we mean that we can draw the solution set of $F_1(x_1, x_2, \dots) = 0, F_2(x_1, x_2, \dots) = 0, \dots$, where the F_i are polynomials.

Theorem 1.2 (Purel[1], 1970s)**.** *For every differentiable semi-algebraic system, there exists a 3-dimensional mechanical instrument which "solves" it.*

This is the case where we find the solution to $\sum a_{i,j,k,\ell} x^i y^j \frac{\partial F}{\partial^k x \partial^\ell y} = 0$. Here, the $a_{i,j,k,\ell} \in \mathbb{R}$.

Proof. Integrate the system as many times as you need to get rid of all the derivatives. Then we get a system of integral equations. We can solve integrals using the planimeter. □

[1] Purel was one of the first famous American female mathematicians.

1.2 Differential analyzers

Around 1915, Vanniver Bush, an engineer at MIT, built a differential analyzer. This was a machine where you put gears in the right places, and if you operate a crank, then it draws the solution to a differential equation.[2] People came from all over the country to use the machine. Around 1925, this became motorized. The machine was able to solve differential equations of order 6.

Claude Shannon was a student at the time, and he was hired to operate the crank. He realized that this was applicable to boolean logic, and wrote his master's thesis on how to compute boolean logic using an electrical computer. This led to the birth of the modern computer.

In the 1940s UCLA bought a massive differential analyzer for $250000. By the time it was made it was already obsolete. It never got used.[3]

[2] Apparently, for harder differential equations, it was a real job to turn the crank.

[3] You can find a hilarious advertisement about it on Professor Pak's website.